Summary of Microeconomic Theory: Lecture 1–7

Lecture 1: Introduction to Microeconomics

- **Definition and Scope**: Economics studies the allocation of scarce resources with trade-offs and opportunity costs.
- **Methodological Foundations**:
  - Maximization (rationality, incentives)
  - Equilibrium (interaction among agents)
  - Efficiency (distinct from distribution)
- **Market Mechanism**: Determination of price and quantity through supply and demand.
- **Analytical Tools**: Focus on price theory and general equilibrium.

Lecture 2: Production and Technology

- Firms transform inputs into outputs under technological constraints.
- **Key Concepts**:
  - Net output, production set Y, input requirement set $V(y)$
  - Isoquant $Q(y)$, production function $f(x)$, transformation function $T(y)$
- **Technological Properties**: Convexity, returns to scale, homogeneity, homotheticity
- **Elasticity of Substitution**: $\sigma = \frac{d\ln(x_2/x_1)}{d\ln|TRS|}$
- **Examples**: Cobb-Douglas, Leontief, CES production functions

Lecture 3: Profit Maximization

- Objective: $\max_x \ pf(x) - wx$
- **FOC**: $p \cdot Df(x^*) = w$
- Isoprofit curves, quasi-concavity

- **Factor Demand and Supply**: $x(p, w)$ and $y(p, w)$, homogeneous of degree 0
- **Comparative Statics**
- **WAPM**: $p^t y^t - w^t x^t \geq p^t y^s - w^t x^s$
- **Profit Function Properties**: Non-decreasing in p, homogeneous degree 1, convex, continuous

Lecture 4: Duality and Cost Minimization

- **Cost Minimization**: $\min_x \ wx$ s.t. $f(x) = y$
- **Lagrangian and FOC**: TRS equals price ratio
- **Conditional Factor Demand**: $x(w, y)$, cost function $c(w, y) = wx(w, y)$
- **Examples**: Leontief (fixed ratio), Linear (corner solution)
- **WACM**: $w^t x^t \leq w^t x^s$ for $y^s \geq y^t$
- **Hotelling's Lemma**: $\frac{\partial \pi}{\partial p_i} = y_i$, $\frac{\partial \pi}{\partial w_i} = -x_i$
- **Envelope Theorem**: $\frac{dM(a)}{da} = \frac{\partial f}{\partial a}\Big|_{x(a)}$

Lecture 5: Cost Function Properties

- **Short-run and Long-run Cost**:
  - STC, SAC, SAVC, SAFC, SMC
  - LTC, LAC, LMC
- **AC and MC**: $MC = AC$ at minimum AC
- **Cost Function Properties**: Non-decreasing, homogeneous degree 1, concave, continuous
- **Shephard's Lemma**: $\frac{\partial c}{\partial w_i} = x_i(w, y)$
- **Duality**: Recover technology from cost under convexity and monotonicity

Lecture 6: Duality Continued

- **Outer Bound from Data**: $V^*(y) = \{x : wx \geq c(w, y), \forall w \geq 0\}$
- **Recovery**: If $V(y)$ is convex and monotonic, then $V(y) = V^*(y)$
- **Conditions for Valid Cost Functions**:
  - Non-negative, homogeneous (deg 1), monotonic, concave
- **Example**: Recover Cobb-Douglas from cost
- **Geometry**: Isoquant slope = price ratio; Isocost slope = input ratio

Lecture 7: Consumer Theory

- **Preference Axioms**: Completeness, transitivity, reflexivity, continuity, monotonicity, local nonsatiation, convexity
- **Utility Function**: Ordinal representation; existence theorem
- **MRS**: $-\frac{\partial u/\partial x_i}{\partial u/\partial x_j}$
- **Utility Maximization**: $\max u(x)$ s.t. $px \leq m$
- **Marshallian Demand**: $x(p, m)$
- **Indirect Utility Function**: $v(p, m)$
- **Expenditure Function**: $e(p, u)$
- **Hicksian Demand**: $h(p, u)$
- **Identities**:
  - $e(p, v(p, m)) = m$, $x(p, m) = h(p, v(p, m))$
- **Roy's Identity**: $x_i(p, m) = -\frac{\partial v/\partial p_i}{\partial v/\partial m}$